Odd spin glueball masses and the Odderon Regge trajectories from the holographic hardwall model

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Abstract

We use the holographic hardwall model to calculate the masses of light glueball states with odd spin and $P = C = -1$ associated with Odderons. Using Dirichlet and Neumann boundary conditions we obtain expressions for the Odderon Regge trajectories consistent with those calculated by other approaches.

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I. INTRODUCTION

Mesons and baryons have their total angular momenta (J) related to the square of their masses (m) through approximate linear functions known as Regge trajectories:

$$J(m^2) \simeq \alpha_0 + \alpha' m^2, \tag{1}$$

where α_0 and α' are constants characteristic of each hadronic branch. Analogously one can find Regge trajectories for odd spin glueballs with $P = C = -1$ which are related to the Odderon.

The Regge trajectories for the Odderon were obtained by Llanes-Estrada, Bicudo and Cotanch [1] using two different methods. The first one is based on a relativistic many-body (RMB) formulation which gives (masses are expressed in GeV throughout this paper):

$$J_{RMB}(m^2) = -0.88 + 0.23m^2. \tag{2}$$

The second method is based on a nonrelativistic constituent model (NRCM) resulting in:

$$J_{NRCM}(m^2) = 0.25 + 0.18m^2. \tag{3}$$

Interesting studies of the Odderon in gauge/string dualities were presented in refs. [2, 3].

In this work we obtain the masses of odd spin glueballs from the holographic hardwall model and derive the corresponding Regge trajectories for the Odderon compatible with the above results.

Since its conception quantum chromodynamics (QCD) has been used as the standard theory to explain the phenomenology of strong interactions. Due to asymptotic freedom, the coupling of strong interactions decreases when the energy of the process increases. This result is obtained using perturbation theory and is valid only for small couplings ($g < 1$). Extrapolating this result to low energies one obtains strong coupling ($g > 1$), outside the perturbative regime. Regge trajectories are an exemple of non-pertubative behavior of strong interactions and so are difficult to explain using QCD.

The AdS/CFT correspondence [4–9] brought new perspectives for string and quantum field theories since it relates $SU(N)$ supersymmetric and conformal Yang-Mills field theory for $N \to \infty$, in flat Minkowski spacetime with $3 + 1$ dimensions, with a string theory in a curved 10 dimensional spacetime, the $AdS_5 \times S^5$ space. In the supergravity approximation

of string theory in this space one can relate both theories through [5, 6]:

$$Z_{CFT}[\varphi_o] = \left\langle \exp\left(\int_{\partial\Omega} d^4x \; \mathcal{O}\varphi_o \right) \right\rangle = \int_{\varphi_o} D\varphi \exp(-I_s(\varphi)), \tag{4}$$

where φ is a non-normalizable supergravity field, $I_s(\varphi)$ is the corresponding on shell supergravity action, φ_o is the value of φ at the boundary $\partial\Omega$ and $\mathcal{O}$ is the associated operator of the conformal field theory (CFT). From this equation, one can obtain 4 dimensional correlation functions, for instance:

$$\langle \mathcal{O}(x)\mathcal{O}(y) \rangle = \left. \frac{\delta Z_{CFT}[\varphi_o]}{\delta\varphi_o(x)\delta\varphi_o(y)} \right|_{\varphi_o=0}. \tag{5}$$

In particular, the scalar glueball 0^{++} is represented by the operator $\mathcal{O}_4 = F^2$ associated with a dilaton in the $AdS_5 \times S^5$ space.

II. ODD SPIN GLUEBALLS MASSES IN THE HARDWALL MODEL

Glueballs are characterized by J^{PC}, where J represents the total angular momentum, P defines how a state behaves under spatial inversion (P-parity) and C shows the behavior of a state under charge conjugation (C-parity).

In this paper we are interested in glueballs in the $P = -1$ and $C = -1$ sector with odd spins $J \geq 1$ which are associated with a particle called the *Odderon*. The concept of the Odderon emerged in the 70's [10], within the context of asymptotic theorems, reappearing later in perturbative QCD [11, 12]. The Odderon have also been linked, for instance, to the color glass condensate [13]. Although the Odderon has not been detected so far, there is some experimental evidence of its existence and it could be regarded as a test of QCD [14]. The Odderon is a bound state of three gluons, without color, which represents a singularity in the complex plane J, close to 1, in the odd-under-crossing amplitude $F_{-}(s,t)$ [15]. For a review see for instance [16].

The AdS/CFT correspondence can not be used directly as a tool for the study of hadrons, because the dual theory is a supersymmetric conformal theory which is very different from QCD. However, it was noticed that the energy E of a process in the $4d$ theory is related to the radial coordinate z in AdS space as

$$E \propto \frac{1}{z}. \tag{6}$$

This motivated the holographic hardwall model proposed by Polchinski and Strassler [17, 18] to calculate the scattering of glueballs in 4-dimensions using a dilaton field in $AdS_5 \times S^5$ space. The works [19, 20] introduced a cut-off at a certain value z_{max} of the z coordinate and considered an AdS slice in the region $0 \leq z \leq z_{max}$. An immediate consequence of introducing a cut-off is the breaking of conformal invariance, so that particles on the 4-dimensional boundary acquire mass. Futhermore, one can associate the size of the AdS slice with the energy scale of QCD:

$$z_{max} = \frac{1}{\Lambda_{QCD}}. \tag{7}$$

The hardwall model assumes an approximate duality between a string theory in an $AdS_5 \times S^5$ space with metric defined by:

$$ds^2 = \frac{R^2}{z^2}(-dt^2 + d\vec{x}^2 + dz^2) + R^2 d\Omega_5^2, \tag{8}$$

where R is the AdS radius, and a pure Yang-Mills theory in four dimensions with symmetry group $SU(N)$ in the large N limit. In this model it is assumed that the AdS/CFT dictionary between supergravity fields in $AdS_5 \times S^5$ space and operators on the 4d boundary, as given by eqs. (4) and (5), still holds after breaking the conformal invariance. This implies that the conformal dimension Δ of an operator $\mathcal{O}$ related to a p-form AdS_5 field with mass m_5 is given by [21] (here and in the following we are disregarding excitations on de S^5 subspace):

$$m_5^2 R^2 = (\Delta - p)(\Delta + p - 4). \tag{9}$$

In particular, the operator that describes the glueball 1^{--} is

$$SymTr(\tilde{F}_{\mu\nu} F^2) \tag{10}$$

with conformal dimension $\Delta = 6$. This operator is associated with the Ramond-Ramond tensor $C_{2,\sigma\lambda}$ described in a single $D3$-brane, by the action [2, 22]:

$$\mathcal{I} = \int d^4x \, \det \left[G_{\sigma\lambda} + \exp^{-\frac{\phi}{2}}(B_{\sigma\lambda} + F_{\sigma\lambda}) \right] + \int d^4x (C_0 F \wedge F + C_2 \wedge F + C_4). \tag{11}$$

From this action one can obtain the equations of motion for the Ramond-Ramond field. With a suitable polarization choice $C_{2,\sigma\lambda}(x,z) = c_{\sigma\lambda}\phi(x,z)$ where $c_{\sigma\lambda}$ is a constant polarization tensor and $\phi(x,z)$ is a scalar field, it can be shown that these equations can be reduced to [23]:

$$\left[z^3 \partial_z \frac{1}{z^3} \partial_z + \eta^{\alpha\beta}\partial_\alpha\partial_\beta - \frac{m_5^2 R^2}{z^2} \right] \phi(x,z) = 0, \tag{12}$$

where $\eta^{\alpha\beta}$ is the 4-dimensional Minkowski metric.

We use a plane wave ansatz in the 4-dimensional space for the 0-form field ϕ

$$\phi(x,z) = A_{\nu,k} \exp^{-ip.x} z^2 J_\nu(u_{\nu,k} z), \tag{13}$$

where $A_{\nu,k}$ is a normalization constant, $\nu = \sqrt{4 + m_5^2 R^2}$ and the discrete modes $u_{\nu,k}$ ($k = 1, 2, 3, ...$) will be calculated by imposing appropriate boundary conditions.

It has been proposed in the literature [24] that the glueball operator with spin ℓ, could be obtained by the insertion of symmetrized covariant derivatives in the operator $\mathcal{O}_4 = F^2$, such that $\mathcal{O}_{4+\ell} = FD_{\{\mu_1 \cdots} D_{\mu_\ell\}} F$ with conformal dimension $\Delta = 4 + \ell$. This approach was used in ref. [25] to calculate the masses of glueball states 0^{++}, 2^{++}, 4^{++}, 6^{++}, etc and to obtain the corresponding Pomeron Regge trajectory.

Here we are going to follow a similar approach for the glueball states 1^{--}, 3^{--}, 5^{--}, 7^{--}, ... The state 1^{--} is described by the operator $\mathcal{O}_6 = SymTr(\tilde{F}_{\mu\nu} F^2)$. Inserting covariant derivatives as described above, one obtains $\mathcal{O}_{6+\ell} = SymTr(\tilde{F}_{\mu\nu} F D_{\{\mu_1 \cdots} D_{\mu_\ell\}} F)$ with $\Delta = 6 + \ell$ satisfying equations similar to (12) and (13) with a shift in the index of the Bessel function $\nu \to \nu = 4 + \ell$, where $\ell = J \geq 1$ is the spin of each state 1^{--}, 3^{--}, 5^{--}, etc.

Following the approach of ref. [25], we impose Dirichlet and Neumann boundary conditions to calculate glueball masses within the hardwall model. For the Dirichlet boundary condition:

$$\phi(z = z_{max}) = 0 \tag{14}$$

one obtains from (13), the following relation:

$$u_{\nu,k} = \frac{\chi_{\nu,k}}{z_{max}} = \chi_{\nu,k} \Lambda_{QCD}; \quad J_\nu(\chi_{\nu,k}) = 0. \tag{15}$$

On the other hand, for the Neumann boundary condition:

$$\partial_z \phi|_{(z=z_{max})} = 0 \tag{16}$$

one gets:

$$(2 - \nu) J_\nu(\xi_{\nu,k}) + (\xi_{\nu,k}) J_{\nu-1} + \xi_{\nu,k} = 0 \tag{17}$$

where

$$u_{\nu,k} = \frac{\xi_{\nu,k}}{z_{max}} = \xi_{\nu,k} \Lambda_{QCD}. \tag{18}$$

Using these boundary conditions we obtain glueball masses in the sector $P = C = -1$. Our results are shown in Table I. We also show for comparison the values for these masses found in the literature [1, 26–31] using other methods.

Table I. Glueball masses for states J^{PC} expressed in GeV, with odd J estimated using the hardwall model with Dirichlet and Neumann boundary conditions. The mass of 1^{--} is used as an input from the isotropic lattice [30, 31]. We also show other results from the literature for comparison.

Models Used	Glueball States					
	1^{--}	3^{--}	5^{--}	7^{--}	9^{--}	11^{--}
Hardwall with Dirichlet b.c.	3.24	4.09	4.93	5.75	6.57	7.38
Hardwall with Neumann b.c.	3.24	4.21	5.17	6.13	7.09	8.04
Relativistic Many Body [1]	3.95	4.15	5.05	5.90		
Non-Relativistic Constituent [1]	3.49	3.92	5.15	6.14		
Wilson Loop [26]	3.49	4.03				
Vacuum Correlator [27]	3.02	3.49	4.18	4.96		
Vacuum Correlator [27]	3.32	3.83	4.59	5.25		
Semirelativistic Potencial [28]	3.99	4.16	5.26			
Anisotropic Lattice [29]	3.83	4.20				
Isotropic Lattice [30, 31]	3.24	4.33				

III. ODDERON REGGE TRAJECTORIES IN THE HARDWALL MODEL

Using the data for odd spin glueballs obtained in the previous section we are going to built up the Regge trajectories for the Odderon.

Using Dirichlet boundary condition and the set of states, 1^{--}, 3^{--}, 5^{--}, 7^{--}, 9^{--}, 11^{--}, we find the following Regge trajectory:

$$J_{Dir.}^{\{1-11\}}(m^2) = -(0.83 \pm 0.40) + (0.22 \pm 0.01)m^2 \,. \tag{19}$$

The errors for the slope and linear coefficient come from the linear fit. The plot relative to this trajectory can be seen in Figure 1. This result is in agreement with that found in [1], with the relativistic many-body Hamiltonian formulation, described by equation (2).

It has been argued in ref. [1] that the state 1^{--} might not be part of the spectrum of the Odderon. To test this possibility we also consider another set of states, 3^{--}, 5^{--}, 7^{--}, 9^{--}, for which we obtain the following Regge trajectory:

$$J_{Dir.}^{\{3-9\}}(m^2) = -(0.63 \pm 0.31) + (0.23 \pm 0.01)m^2. \tag{20}$$

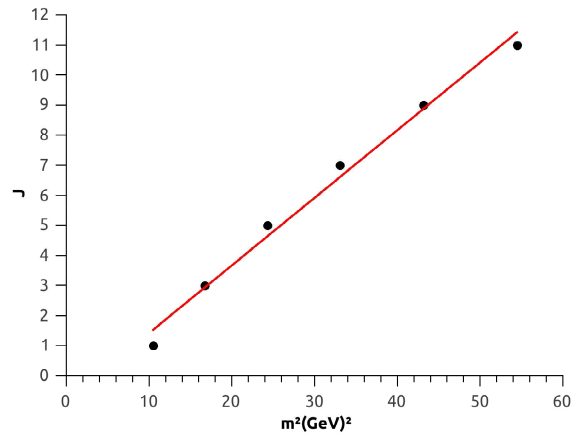

Figure 1. Glueball masses (dots) for the states 1^{--}, 3^{--}, 5^{--}, 7^{--}, 9^{--}, 11^{--} from the holographic hardwall model using Dirichlet boundary condition, eqs. (14) and (15). We also plot an approximate linear Regge trajectory, corresponding to eq. (19), representing the Odderon.

This result is also consistent with the Regge trajectory for Odderon, equation (2).

Now using Neumann boundary condition and the set of states, $1^{--}, 3^{--}, 5^{--}, 7^{--}, 9^{--}, 11^{--}$, we find the following Regge trajectory:

$$J_{Neu.}^{\{1-11\}}(m^2) = -(0.29 \pm 0.42) + (0.18 \pm 0.01)m^2. \tag{21}$$

The plot relative to this trajectory can be seen in Figure 2.

We also consider here the possibility of excluding the state 1^{--} from the spectrum of the Odderon. For the set of states $3^{--}, 5^{--}, 7^{--}, 9^{--}, 11^{--}$, we find the following Regge trajectory

$$J_{Neu.}^{\{3-11\}}(m^2) = (0.34 \pm 0.37) + (0.17 \pm 0.01)m^2. \tag{22}$$

This result is in agreement with that found in [1], with the nonrelativistic constituent model, equation (3).

IV. CONCLUSIONS

In this work we obtained odd spin glueball masses in the sector $P = C = -1$ using the holographic hardwall model with Dirichlet and Neumann boundary conditions. These glueball masses lie in approximate linear Regge trajectories compatible with results for the Odderon, both in the relativistic many-body as well in the non-relativistic constituent model

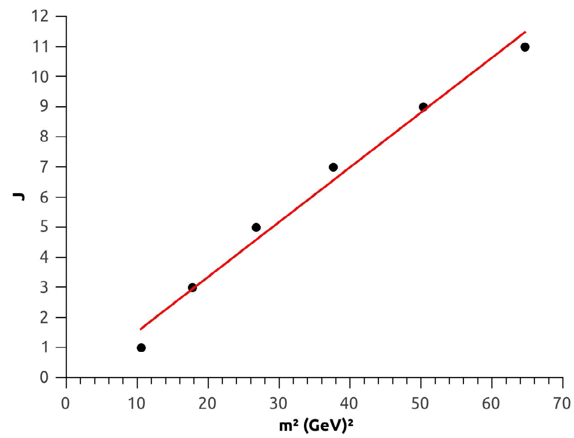

Figure 2. Glueball masses (dots) for the states 1^{--}, 3^{--}, 5^{--}, 7^{--}, 9^{--}, 11^{--} from the holographic hardwall model using Neumann boundary condition, eqs. (16) and (18). We also plot an approximate linear Regge trajectory, corresponding to eq. (21), representing the Odderon.

presented in ref. [1]. The present analysis gives support to the conclusion of ref. [1] about the general properties of the Odderon Regge trajectories, i.e., a low intercept and a slope similar to that of the Pomeron.

Some aspects of the holographic approach for the Odderon Regge trajectories remain open. In our approach, we used Dirichlet and Neumann boundary conditions in the hardwall model obtaining results compatible with those of ref. [1]. The hardwall model was used before to obtain the Regge trajectory for the Pomeron in ref. [25]. In that work it was possible to conclude that Neumann boundary condition was more appropriate than the Dirichlet boundary condition by comparison with experimental data. Here in this work, it is not possible to reach a similar conclusion about boundary conditions because there is no clear experimental data for the Odderon Regge trajectories.

Another open question in the Odderon Regge trajectories regards the state 1^{--}. It was argued in ref. [1] that the glueball state 1^{--} does not belong to the Odderon Regge trajectory. However, from our analysis it is not possible to reach this conclusion since we have found trajectories compatible with Odderon including the state 1^{--} (eqs. (19) and (21)) as well excluding it (eqs. (20) and (22)).

As a final remark, let us comment on our choice for the holographic model to obtain glueball masses and the Odderon Regge trajectories. This model is very interesting since masses

can be obtained from the zeros of the corresponding Bessel functions. However, it is well known that holographic hardwall model leads to asymptotic non linear Regge trajectories for very high states. Nevertheless, for light states, as discussed in this work, approximate linear Regge trajectories were found. In this regard, it wil be interesting to investigate the glueball masses in the $P = C = -1$ sector within other holographic approaches, as the softwall model [32, 33] which is known to provide linear Regge trajectories. We leave this study for future work.

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